

# Distributed Altitude and Attitude Estimation from Multiple Distance Measurements

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**Abstract**—This paper describes a generalized method for computing the altitude and attitude of a rigid body with respect to an inertial frame using a set of distance measurements obtained from a sensor network. In the case where all sensors are centrally measurable, a linear-optimal estimate is obtained. This method is used as a way for estimating altitude and attitude of the Distributed Flight Array, a modular multi-propeller flying vehicle where each module in the array obtains its own distance measurement and coordinates with its immediate neighbour(s) actions for flight. To account for communication bandwidth constraints, a scalable, distributed scheme is presented where each module shares local information. In the limit of sharing information, each module asymptotically computes the linear-optimal altitude and attitude estimate.

## I. INTRODUCTION

One of the key challenges in designing a vertical take-off and landing vehicle is obtaining relatively good altitude and attitude estimates for controlled flight [1], [2]. Various methods currently exist [3]–[5] that address the challenge of estimating the altitude and attitude of a vehicle on board. With the advent of distributed sensor networks [6], [7] and modular robotics [8], [9], a new degree of freedom in obtaining state estimates is available to us – shared information.

This paper presents a generalized, straightforward approach not often seen in the context of flight for estimating the altitude and attitude of a body coordinate frame with respect to an inertial coordinate frame of reference. The idea is to use an array of distance measuring sensors that are attached to known positions on a rigid body [10]. Given the distance to the ground measured by each sensor, it is possible to compute the altitude and attitude of the body, the precision of which increases as a function of the number of sensors.

This method is directly applied to a modular flying robot, in this case the Distributed Flight Array (DFA) [11], see Fig. 1. The homogeneous modules of the DFA are outfitted with an assortment of sensors, including a distance measurement sensor, and each is capable of communicating with its immediate neighbours. Implementation of the proposed method, which is nothing more than solving a least squares problem on a centralized system, adds a new twist as the method must be scalable and carried out in a distributed manner. Borrowing results obtained from consensus literature [12], [13] the method is modified such that in the limit of sharing information between neighbouring modules, the altitude and attitude estimates approach those obtainable in a centralized system.

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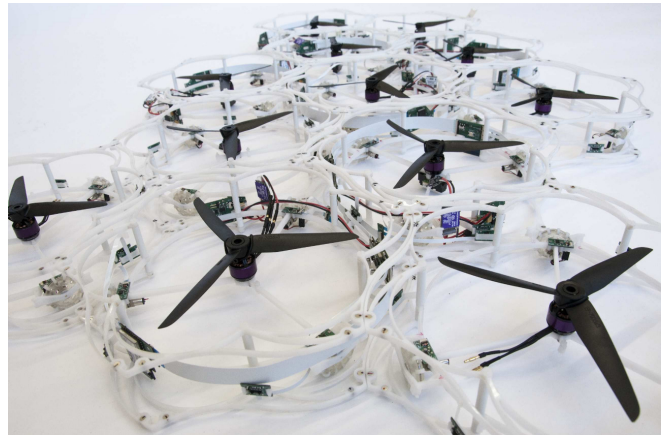


Fig. 1. The Distributed Flight Array is a modular multi-propeller vehicle that can fly in ad-hoc configurations; it is an example of a distributed control and estimation system.

This method of attitude estimation can then be fused with other forms of attitude sensing and estimation such as integrated angular rates from rate-gyros, which have good short-term characteristics but suffer from long-term drift. Sensor fusion can be accomplished using, for example, a Kalman filter or a complimentary filter [3], [14].

This work begins in Sec. II by showing how distance measurements from sensor to ground taken from anywhere on a rigid body are related to the body's altitude and attitude with respect to an inertial frame. In Sec. III it is shown that altitude and attitude of the rigid body can in general be estimated from an arbitrary number of sensors placed in arbitrary positions through a least squares approach. This technique is applied to the DFA in Sec. IV, and modified to a scalable, distributed scheme which results in estimates that in the limit of sharing information approach those obtainable in a centralized system. Experimental results are presented and compared to results obtained from simulation in Sec. V.

## II. KINEMATIC MODEL

A kinematic model that demonstrates the dependency of distance measurements on altitude and attitude of a spatial geometry with respect to an inertial coordinate frame will be developed in this section. Although this model can be generalized, the DFA is used here as an example to convey better understanding, see Fig. 2.

The  $X_I, Y_I$ -plane of the inertial coordinate frame ( $I$ ) coincides with the ground, which is assumed to be perpendicular to the gravity field and flat.

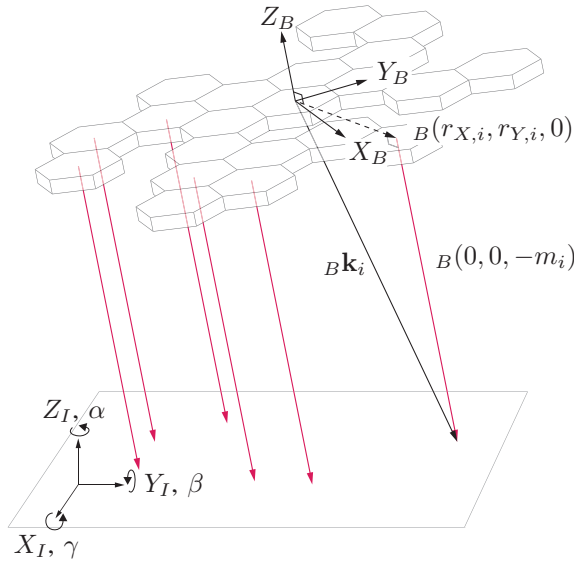


Fig. 2. The DFA's body coordinate frame ( $B$ ) is rotated by roll  $\gamma$ , pitch  $\beta$ , yaw  $\alpha$  about the  $X_I$ ,  $Y_I$ , and  $Z_I$ -axis of the inertial frame ( $I$ ), respectively. The  $X_I, Y_I$ -plane of the inertial frame coincides with the ground. The position of sensor  $i$  is denoted by  ${}_B(r_{X,i}, r_{Y,i}, 0)$ . The distance vector  ${}_B(0, 0, -m_i)$  of sensor  $i$  is parallel to the  $Z_B$ -axis and coincides with the ground at point  ${}_B\mathbf{k}_i = {}_B(r_{X,i}, r_{Y,i}, -m_i)$ .

Assume the DFA to be a rigid body described by the body coordinate frame ( $B$ ), which is located at the center of mass and is aligned with its principal axes of rotation. The body frame's altitude with respect to the inertial frame is denoted by  $z$  and its orientation with respect to the inertial frame can be described through a set of fixed rotations roll  $\gamma$ , pitch  $\beta$ , and yaw  $\alpha$  about the  $X_I$ ,  $Y_I$ , and  $Z_I$ -axis of the inertial frame, respectively [15]. In the following, position with respect to the body frame and inertial frame is denoted by  ${}_B(\cdot)$  and  ${}_I(\cdot)$ .

Without loss of generality, each sensor  $i = \{1, \dots, N\}$  is assumed to be located in the  $X_B, Y_B$ -plane of the body frame at the respective known position  ${}_B(r_{X,i}, r_{Y,i}, 0)$ . The distance of sensor  $i$  to the ground in the direction of the  $Z_B$ -axis is denoted as  $m_i$ . The distance vector coincides with the ground at point  ${}_B\mathbf{k}_i = {}_B(r_{X,i}, r_{Y,i}, -m_i)$ , which results from adding sensor  $i$ 's position and the respective distance.

Any point  ${}_B\mathbf{k}_i$  that lies in the  $X_I, Y_I$ -plane of the inertial frame satisfies the Hesse Normal Form [16]

$${}_B\mathbf{n}^T {}_B\mathbf{k}_i - z = 0, \quad (1)$$

where  ${}_B\mathbf{n}$  is the unit normal of the plane with respect to the body frame and may be obtained from the coordinate transformation

$${}_B\mathbf{n} = {}_B^I\mathbf{R}^T {}_I\mathbf{n}, \quad (2)$$

with  ${}_B^I\mathbf{R}$  being a rotation matrix that maps a vector from the body frame to the inertial frame and  ${}_I\mathbf{n} = (0, 0, -1)$  denotes the unit normal of the plane with respect to the inertial frame. According to the convention of the Hesse Normal Form, the unit normal  ${}_I\mathbf{n}$  points away from the body coordinate frame to the plane of interest.

The rotation matrix  ${}_B^I\mathbf{R}$  results from a series of matrix multiplications,

$${}_B^I\mathbf{R} = \mathbf{R}_z(\alpha)\mathbf{R}_y(\beta)\mathbf{R}_x(\gamma), \quad (3)$$

and in consequence, the unit normal  ${}_B\mathbf{n}$  with respect to the body frame is a function of roll  $\gamma$  and pitch  $\beta$ ,

$${}_B\mathbf{n} = \begin{bmatrix} \sin(\beta) \\ -\cos(\beta)\sin(\gamma) \\ -\cos(\beta)\cos(\gamma) \end{bmatrix}. \quad (4)$$

Making the appropriate substitutions yields an equation relating distance  $m_i$  to altitude and attitude of the DFA,

$$\begin{bmatrix} \sin(\beta) \\ -\cos(\beta)\sin(\gamma) \\ -\cos(\beta)\cos(\gamma) \end{bmatrix}^T \begin{bmatrix} r_{X,i} \\ r_{Y,i} \\ -m_i \end{bmatrix} - z = 0. \quad (5)$$

Rearranging this expression in terms of  $m_i$  yields

$$m_i = \frac{z + r_{Y,i}\cos(\beta)\sin(\gamma) - r_{X,i}\sin(\beta)}{\cos(\beta)\cos(\gamma)}, \quad (6)$$

where distance  $m_i$  of sensor  $i$  to the ground is nonlinearly dependent on the states altitude  $z$ , roll  $\gamma$ , and pitch  $\beta$  of the DFA and sensor  $i$ 's position  ${}_B(r_{X,i}, r_{Y,i}, 0)$ . Thus, the distance of sensor  $i$  to the ground is independent of yaw  $\alpha$ . In the context of this work, terminology is abused by using the term attitude to refer to just roll and pitch of a body, which is sometimes denoted as tilt.

The nominal flight conditions of the DFA are around hover and therefore the deviation from hover will be relatively small. Thus, when linearised about hover the relationship between distance  $m_i$  and states  $z$ ,  $\gamma$ , and  $\beta$  reads as

$$m_i = z + r_{Y,i}\gamma - r_{X,i}\beta. \quad (7)$$

Intuitively, the distance of a sensor to the ground increases with the altitude of the DFA. In addition, the further away a sensor is located along the  $X_B$  and  $Y_B$ -axis of the body frame, the more its distance to the ground is affected by pitch and roll, respectively.

The kinematic model relating distance to states denoted by  $\mathbf{x} = (z, \gamma, \beta)$  can be more compactly written as

$$m_i = \mathbf{c}_i\mathbf{x}, \quad (8)$$

where row vector  $\mathbf{c}_i = (1, r_{Y,i}, -r_{X,i})$  contains information pertaining to the position of sensor  $i$  with respect to the body frame and maps altitude, roll, and pitch to a distance.

### III. ALTITUDE AND ATTITUDE ESTIMATION

At least three distance measurements coming from sensors that are laterally unaligned in the  $X_B, Y_B$ -plane of the body frame are sufficient to compute altitude  $z$ , roll  $\gamma$ , and pitch  $\beta$ . The noisy distance measurement  $\hat{m}_i$  of sensor  $i$  is modelled as

$$\hat{m}_i = m_i + v_i, \quad (9)$$

where  $m_i$  is the sensor's distance to the ground and  $v_i$  models zero mean noise with variance  $\sigma^2$ . The noises  $v_i$  are assumed to be independent and identically distributed.

Under the assumption of small angles, the unknown states  $\mathbf{x} = (z, \gamma, \beta)$  can be computed from a linear set of equations. This is accomplished by combining sensor positions with sensor measurements, obtained simultaneously,

$$\hat{\mathbf{y}} = \mathbf{C}\mathbf{x} + \mathbf{v}, \quad (10)$$

where  $\hat{\mathbf{y}} = (\hat{m}_1, \dots, \hat{m}_N)$  denotes the combined measurement,  $\mathbf{C}$  contains information about the position of all sensors with respect to the body frame,

$$\mathbf{C} = \begin{bmatrix} 1, & r_{Y,1}, & -r_{X,1} \\ \vdots & \vdots & \vdots \\ 1, & r_{Y,N}, & -r_{X,N} \end{bmatrix}, \quad (11)$$

and  $\mathbf{v} = (v_1, \dots, v_N)$  is the combined sensor noise. The covariance matrix  $\Sigma$  of  $\mathbf{v}$  is written as  $\sigma^2 \mathbf{I}_{N \times N}$ .

When there are three distance measurements from laterally unaligned sensors the linear set of equations is determined and the solution is trivial.

For more than three sensors the set is overdetermined and the maximum-likelihood state estimate  $\hat{\mathbf{x}}$  can be obtained by solving a least squares problem on a centralized system [17]. This is the linear-optimal estimate and reads as

$$\hat{\mathbf{x}} = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \hat{\mathbf{y}}, \quad (12)$$

where  $\mathbf{C}$  is assumed to have full column rank such that  $\mathbf{C}^T \mathbf{C}$  is invertible, which is the case if the sensors are laterally unaligned.

The expected value of the state estimate is unbiased and therefore given by

$$\mathbb{E}[\hat{\mathbf{x}}] = \mathbf{x}. \quad (13)$$

In general and under the assumption of identically distributed noise, the covariance matrix  $\mathbf{Q}$  of the estimation error  $(\hat{\mathbf{x}} - \mathbf{x})$  can be written as

$$\begin{aligned} \mathbf{Q} &= \mathbb{E}[(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^T] \\ &= (\mathbf{C}^T \mathbf{C})^{-1} \sigma^2. \end{aligned} \quad (14)$$

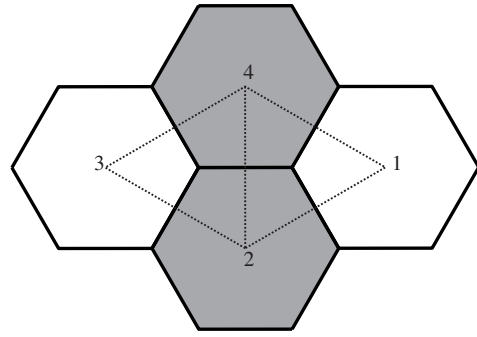
Therefore, the covariance matrix is simply a scaled variance of the distance measurements. Intuitively, the covariance decreases with the number of sensors and the further they are spatially distributed. This can be readily seen by considering the case where the sensors are symmetrically distributed with respect to the body frame. Then, the covariance matrix can be written as

$$\mathbf{Q} = \begin{bmatrix} N & 0 & 0 \\ 0 & \sum_{i=1}^N r_{Y,i}^2 & 0 \\ 0 & 0 & \sum_{i=1}^N r_{X,i}^2 \end{bmatrix}^{-1} \sigma^2. \quad (15)$$

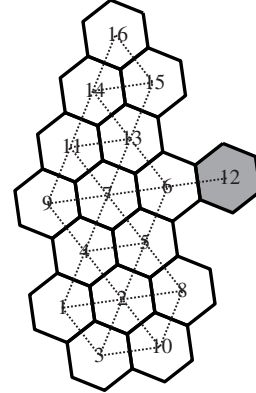
Thus, the variance of the estimation error in altitude scales with the number of sensors, whereas the variance of the estimation error in attitude scales with the further they are away from the center of mass.

#### IV. ALTITUDE AND ATTITUDE ESTIMATION ON THE DISTRIBUTED FLIGHT ARRAY

The DFA is modelled as a sensor network which is described by a fixed, undirected, and connected graph  $\mathcal{G} = (\mathcal{E}, \mathcal{V})$ , see Fig. 3. Each node  $i$  of the node set  $\mathcal{V} = \{1, \dots, N\}$  represents a module which is equipped with a sensor to obtain distance measurements. The edge set  $\mathcal{E} \subset \{\{i, j\} | i, j \in \mathcal{V}\}$  represents communication links where each edge  $\{i, j\}$  is an unordered pair of distinct nodes. The neighbour set of module  $i$  is denoted by  $\mathcal{N}_i = \{j | \{i, j\} \in \mathcal{E}\}$  and  $d_i = |\mathcal{N}_i|$  denotes the degree of a module. Due to geometric constraints of a module, the degree can take values  $d_i \in \{1, \dots, 6\}$ . Each module can only communicate with its immediate neighbours  $j \in \mathcal{N}_i$ .



(a) Module-2 and module-4 are fully connected to all other modules in the array, whereas module-1 and module-3 are only connected to module-2 and module-4.

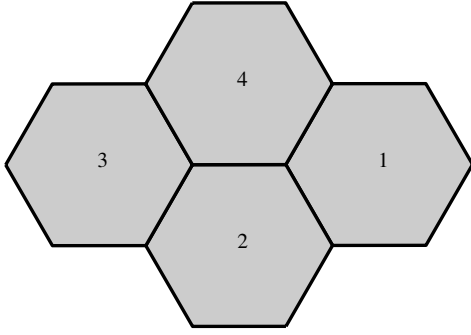


(b) Module-12 has only one neighbour. Its degree  $d_{12}$  therefore equals one and it cannot obtain a state estimate.

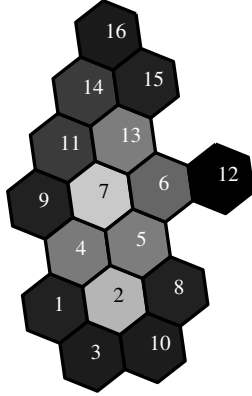
Fig. 3. Two configurations of the Distributed Flight Array: (a) 4-module array, and (b) 16-module array. The nodes of the network, which represent DFA modules, are denoted by the respective module identifier ( $\{1, 2, 3, 4\}$  for the 4-module array and  $\{1, \dots, 16\}$  for the 16-module array). The dotted lines denote the possible inter-module communication links.

##### A. Decentralized State Estimation

One way to use the altitude and attitude estimation method described in Sec. III is for each module to flood the network with its distance measurement. Each module in this case will



(a) The estimation error covariance matrix  $\mathbf{Q}_i$  of each module is equivalent to  $\mathbf{Q}$ .



(b) The two-norm of  $\mathbf{Q}_{12}$  for module 12 is not defined as it has only one neighbour. The estimation error covariance matrix  $\mathbf{Q}_i$  of module-2 and module-7 is closest to the estimation error covariance matrix  $\mathbf{Q}$ , yet still three times as large.

Fig. 4. The colour spectrum illustrates the ratio  $\frac{\|\mathbf{Q}_i\|_2}{\|\mathbf{Q}\|_2}$  between the estimation error covariance matrix  $\mathbf{Q}_i$  of module  $i$  and estimation error covariance matrix  $\mathbf{Q}$  which is obtained using all distance measurements from the array in a centralized system. A lighter grey indicates a closer relation between both covariance matrices in the terms of the two-norm.

eventually receive all measurements in the array and can thus implement the previously described procedure directly. Although this may be feasible for small networks, this approach is not scalable for larger networks due to communication bandwidth and on-board memory storage constraints, both of which scale with the size  $N$  of the network.

A straightforward and scalable approach is for module  $i$  to estimate the states in a decentralized manner by combining only distance measurements of its immediate neighbours. For this, module  $i$  must have at least two neighbours such that together they are laterally unaligned. Modules which do not fulfil these criteria cannot estimate altitude and attitude with the described procedure. In this case, state estimates of modules which fulfil the criteria are propagated to them.

The combined measurement  $\hat{\mathbf{y}}_i \in \mathbb{R}^{(d_i+1) \times 1}$  of module  $i$  and its neighbours  $j \in \mathcal{N}_i$  may be written as

$$\hat{\mathbf{y}}_i = \mathbf{C}_i \mathbf{x} + \mathbf{v}_i, \quad (16)$$

where  $\mathbf{C}_i \in \mathbb{R}^{(d_i+1) \times 3}$  is composed of position information

$\mathbf{c}_i$  and  $\mathbf{c}_j$  corresponding to module  $i$  and its neighbours  $j$ ; similarly for the combined noise  $\mathbf{v}_i \in \mathbb{R}^{(d_i+1) \times 1}$ . The state estimate  $\hat{\mathbf{x}}_i$  of module  $i$  can then be obtained from (12) where  $\mathbf{C}_i$  replaces  $\mathbf{C}$ . Recall that this expression is only well-conditioned if module  $i$  and its neighbours are laterally unaligned.

The estimation error covariance matrix of module  $i$  is given by

$$\begin{aligned} \mathbf{Q}_i &= (\mathbf{C}_i^T \mathbf{C}_i)^{-1} \sigma^2 \quad (17) \\ &= \begin{bmatrix} d_i + 1 & \sum_k r_{Y,k} & -\sum_k r_{X,k} \\ \sum_k r_{Y,k} & \sum_k r_{Y,k}^2 & -\sum_k r_{Y,k} r_{X,k} \\ -\sum_k r_{X,k} & -\sum_k r_{Y,k} r_{X,k} & \sum_k r_{X,k}^2 \end{bmatrix}^{-1} \sigma^2, \quad (18) \end{aligned}$$

where  $k \in \{i \cup j | j \in \mathcal{N}_i\}$  is the set of module  $i$  and its immediate neighbours  $j$ . It is readily seen that  $\mathbf{Q}_i$  depends on degree  $d_i$  of module  $i$  as well as the position of module  $i$  and its neighbours  $j$ . Thus, the estimation error covariance matrix varies across the array and is in general greater in terms of the two-norm than the estimation error covariance matrix  $\mathbf{Q}$ , which is obtained using all distance measurements from the array centrally, see Fig. 4.

The state estimates obtained by the modules in the array with at least two neighbours, at any given instance of time, vary spatially across the array since each module combines different distance measurements. For the DFA, a decoupled control strategy was developed where each module generates a control input depending on its position with respect to the body frame and its state estimate. Thus, spatially varying state estimates induce varying and uncoordinated control inputs of the decentralized controller which causes undesirable shear forces [11].

One might attempt to reduce spatial variance by having the modules share their state estimates  $\hat{\mathbf{x}}_i$  between their immediate neighbours and perform average consensus until their estimates asymptotically converge to the average  $\bar{\mathbf{x}}$ ,

$$\bar{\mathbf{x}} = \frac{1}{N} \sum_{i=1}^N \hat{\mathbf{x}}_i = \frac{1}{N} \sum_{i=1}^N (\mathbf{C}_i^T \mathbf{C}_i)^{-1} \mathbf{C}_i^T \hat{\mathbf{y}}_i. \quad (19)$$

It can be shown that generally the covariance matrix of estimation error  $(\bar{\mathbf{x}} - \mathbf{x})$  in this approach is greater compared to the covariance matrix of estimation error  $(\hat{\mathbf{x}} - \mathbf{x})$  where the linear-optimal state estimate  $\hat{\mathbf{x}}$  is obtained by (12) using all distance measurements from the array directly in a centralized system.

## B. Distributed State Estimation

In order to yield the linear-optimal state estimate  $\hat{\mathbf{x}}$  on each module in the array, a scalable, distributed method following the results of [12] is presented which in the limit of sharing information between neighbouring modules approaches the linear-optimal state estimate. This approach is applicable to all modules in the array, regardless of their degree.

The linear least squares estimation in (12) can be rewritten as a summation of the square of the position information vectors  $\mathbf{c}_i \in \mathbb{R}^{1 \times 3}$  and a summation of measurements  $\hat{m}_i$  scaled by the position information vectors,

$$\hat{\mathbf{x}} = \left( \sum_{i=1}^N \mathbf{c}_i^T \mathbf{c}_i \right)^{-1} \sum_{i=1}^N \mathbf{c}_i^T \hat{m}_i. \quad (20)$$

This can be broken down into two separate average consensus problems. The idea behind this procedure is that each module computes in the limit the average of  $\mathbf{c}_i^T \mathbf{c}_i$  denoted by  $\bar{\mathbf{P}} \in \mathbb{R}^{3 \times 3}$  and  $\mathbf{c}_i^T \hat{m}_i$  denoted by  $\bar{\mathbf{q}} \in \mathbb{R}^{3 \times 1}$  of all modules  $N$ . From this, the linear-optimal state estimate  $\hat{\mathbf{x}}$  reads as

$$\hat{\mathbf{x}} = \bar{\mathbf{P}}^{-1} \bar{\mathbf{q}}, \quad (21)$$

where

$$\bar{\mathbf{P}} = \frac{1}{N} \sum_{i=1}^N \mathbf{c}_i^T \mathbf{c}_i \quad (22)$$

$$\bar{\mathbf{q}} = \frac{1}{N} \sum_{i=1}^N \mathbf{c}_i^T \hat{m}_i. \quad (23)$$

In order to formulate (22) and (23) in an iterative scheme, each module  $i$  maintains local information  $(\mathbf{P}_i(k), \mathbf{q}_i(k))$  and exchanges this with its immediate neighbours  $j \in \mathcal{N}_i$ . The iterative procedure is a weighted average of module  $i$ 's information with the information of its neighbours,

$$\mathbf{P}_i(k+1) = w_{ii} \mathbf{P}_i(k) + \sum_{j \in \mathcal{N}_i} w_{ij} \mathbf{P}_j(k) \quad (24)$$

$$\mathbf{q}_i(k+1) = w_{ii} \mathbf{q}_i(k) + \sum_{j \in \mathcal{N}_i} w_{ij} \mathbf{q}_j(k). \quad (25)$$

In the first iteration  $k=0$  each module initializes its local information as

$$\mathbf{P}_i(0) = \mathbf{c}_i^T \mathbf{c}_i \quad \text{and} \quad \mathbf{q}_i(0) = \mathbf{c}_i^T \hat{m}_i. \quad (26)$$

The weights  $w$  chosen in this work are called Metropolis weights [12] and are defined as

$$w_{ij} = \begin{cases} \frac{1}{1 + \max\{d_i, d_j\}} & \text{if } j \in \mathcal{N}_i, \\ 1 - \sum_{k \in \mathcal{N}_i} w_{ik} & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases} \quad (27)$$

The weight  $w_{ij}$  which corresponds to the edge between module  $i$  and its neighbour  $j$  is inversely proportional to the maximum degree of both modules. The self-weights  $w_{ii}$  are then chosen such that convergence and unbiased intermediate results  $\hat{\mathbf{x}}_i(k) = \mathbf{P}_i(k)^{-1} \mathbf{q}_i(k)$  are guaranteed [18]. The intermediate results can be obtained if matrix  $\mathbf{P}_i(k)$  is invertible. It can be shown for the DFA that  $\mathbf{P}_i(k)$  is singular at least during the initial step  $\mathbf{P}_i(0)$ . As the number of iterations  $k \rightarrow \infty$ ,  $\mathbf{P}_i(k)$  and  $\mathbf{q}_i(k)$  converge to

$$\lim_{k \rightarrow \infty} \mathbf{P}_i(k) = \bar{\mathbf{P}} \quad (28)$$

$$\lim_{k \rightarrow \infty} \mathbf{q}_i(k) = \bar{\mathbf{q}}, \quad (29)$$

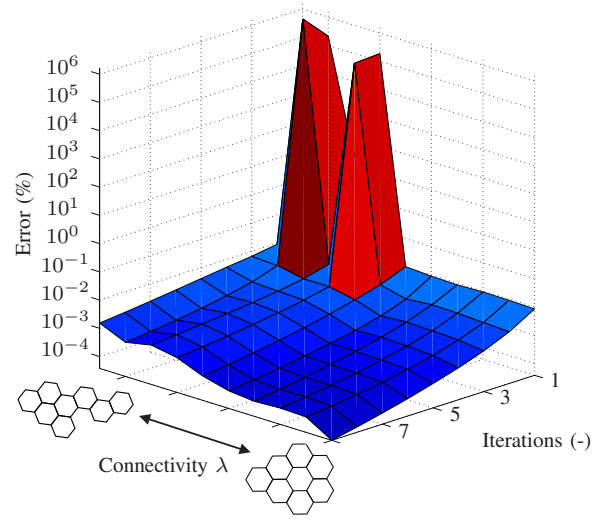


Fig. 5. The maximum error  $\epsilon = \max(\|\hat{\mathbf{x}}_i - \hat{\mathbf{x}}\|_2)$  depends on the connectivity  $\lambda$  of the array and consensus iterations  $k$ . Ten random configurations of an 8-module array are used as an example. The error decreases with the number of iterations and is smaller for densely packed array configurations. For certain configurations where some modules have only one neighbour the first and second iteration yield estimates with errors of magnitude  $10^6$  % as matrix  $\mathbf{P}_i$  is singular during these iterations.

such that each module asymptotically converges to an estimate  $\hat{\mathbf{x}}_i$  which is in the limit equivalent to  $\hat{\mathbf{x}}$ ,

$$\lim_{k \rightarrow \infty} \hat{\mathbf{x}}_i(k) = \lim_{k \rightarrow \infty} \mathbf{P}_i(k)^{-1} \mathbf{q}_i(k) = \hat{\mathbf{x}}. \quad (30)$$

Intuitively, the variance of the local state estimates across the array decreases with the number of iterations, see Fig. 5. The number of iterations needed to guarantee a certain convergence of the modules' estimates  $\epsilon = \max(\|\hat{\mathbf{x}}_i - \hat{\mathbf{x}}\|_2)$  depends on the algebraic connectivity of the network. The connectivity metric is defined as the second smallest Eigenvalue  $\lambda$  of the normalized network Laplacian matrix  $L \in \mathbb{R}^{N \times N}$  which represents the graph of a network [13].

## V. EXPERIMENTS

To show feasibility of altitude and attitude estimation from a set of distance measurements, the proposed estimation scheme of Section III was implemented on a 4-module array and tested in an experiment. To extend the algorithm to larger arrays, a MATLAB simulation was developed.

### A. Hardware Description

A DFA module resembles a hexagon, see Fig. 6, and incorporates custom-designed electronics which were made to meet all the on-board sensing, communication, and computation requirements. Each module comes equipped with an infrared (IR) distance sensor (Sharp GP2Y0AXX series). These commonly used low-cost sensors produce a non-linear output voltage that is a function of the distance being measured. For experiments, a particular model of the sensor was used that is capable of measuring a distance between 20cm and 80cm.

The bandwidth of these sensors is limited to approximately 20Hz. In terms of communication, each module is capable of

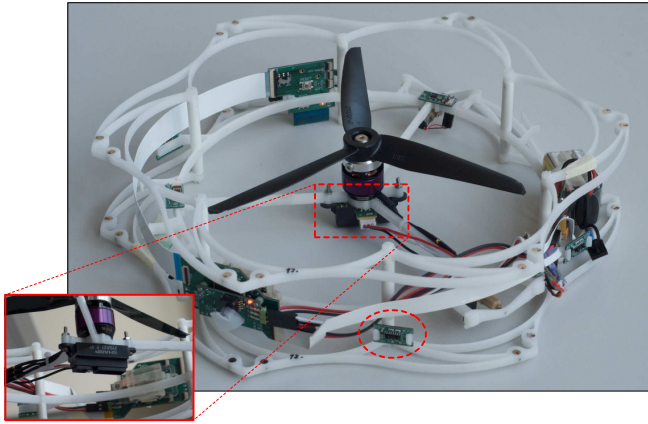


Fig. 6. The inset image in the lower-left corner shows an infrared distance sensor that is mounted in the DFA module's geometric center below the motor and propeller. The encircled part in the figure shows the communication unit on one side of the module that enables inter-module communication.

bi-directional, half-duplex communication with its immediate neighbours. This is accomplished using IrDA transceivers attached to each of the six sides of a modules, and they are interfaced via a UART peripheral capable of operating at 115.2 kbps. A single ARM7 core microcontroller handles all of the required peripheral interfaces and computation needed for estimation and control.

### B. Sensor Calibration

Due to the nonlinear output behaviour and surface dependent response, the sensors were calibrated before the experiment. The function that relates the sampled output voltage  $V_i$  of sensor  $i$  to a distance  $m_i$  is modelled as

$$m_i = a_i + \frac{b_i}{V_i} + \frac{c_i}{V_i^2}, \quad (31)$$

where the coefficients  $a_i, b_i, c_i$  are determined by sampling the output voltage at known distances and solving a linear least squares problem, see Fig. 7.

### C. Results

For the experiment, a 4-module array was mounted in a test bed that enabled fixing the array to a known altitude and attitude. Since two of the modules in the chosen configuration, see Fig. 3 (a) are fully connected to all other modules, they are able to combine all distance measurements and compute the linear-optimal estimate directly. The proposed algorithm was implemented on one of these modules which computed the state estimate online and sent it to a groundstation computer where the estimates were stored for analysis.

The experiment yields quantitatively good results, see Fig. 8. The error between reference angles  $\gamma, \beta$  and estimated attitude is in the range of  $\pm 3^\circ$  which may be due to poor calibration of the IR distance sensors and inaccuracies of the test bed. The error between reference altitude  $z$  and its estimate is in the range of  $\pm 1$  cm.

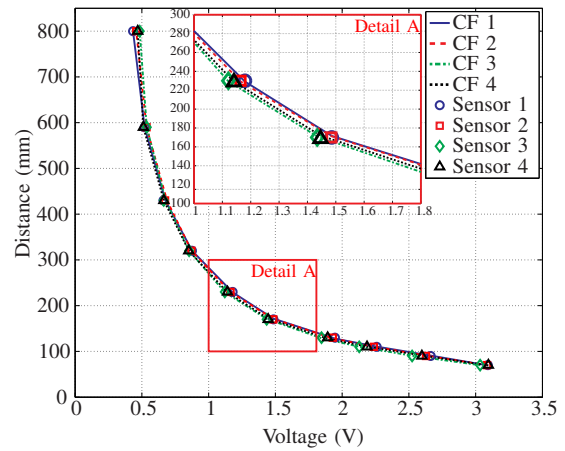


Fig. 7. For calibration, the sensors are placed at 10 different distances to the ground. The output voltage of each sensor at the respective distance is sampled and averaged. A polynomial curve fit (CF) is then fitted through the reference points. Experiments have shown that the sensors work well, even below the specified distance as low as 7cm.

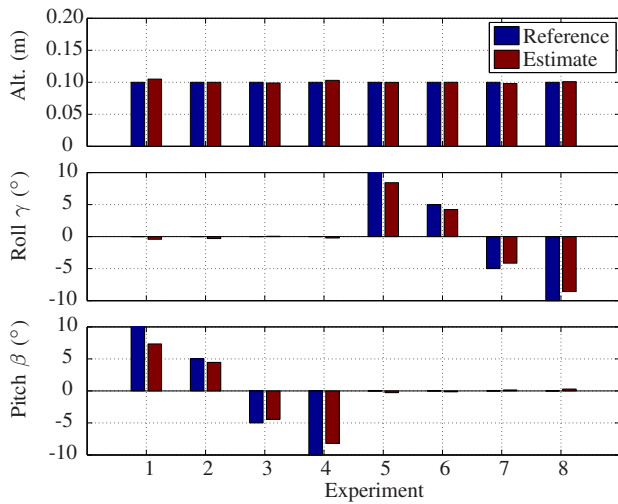
The result of the experiment was used to validate a simulation model of the DFA, which assists in determining the performance of the proposed state estimation technique for larger arrays. For this, the standard deviation of the state estimates obtained from the experiment was compared to the standard deviation of the simulated state estimates. After validation of the simulation model, random configurations with varying array sizes were simulated to demonstrate the benefits of larger arrays. As expected, the results show that the standard deviation of the state estimates decreases with the size of the array, see Fig. 9.

## VI. CONCLUSIONS

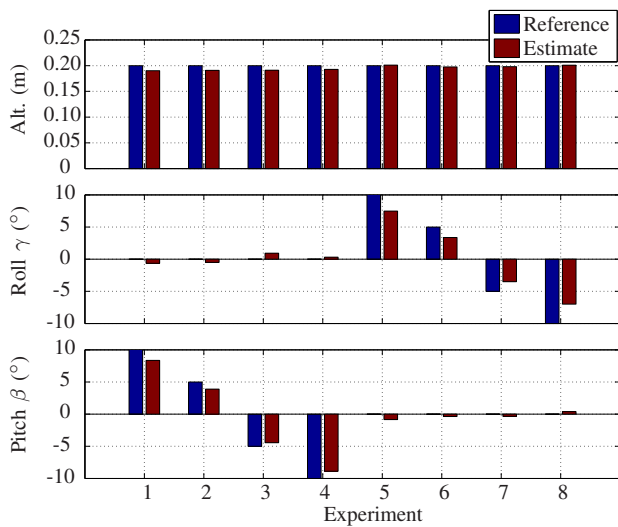
In this work, a generalized method was presented which yields a linear-optimal altitude and attitude estimate of a rigid body with respect to an inertial frame by combining distance measurements obtained from a sensor network. In the case of the DFA, where each module may only exchange information with its immediate neighbours, this approach is not scalable due to limited communication bandwidth. A scalable approach was suggested in which modules only combine distance measurements of their immediate neighbours. However, the resulting state estimates vary spatially across the array, which is undesirable in the case of the DFA's decentralized controller. Instead, the approach was formulated as a distributed averaging problem, where a module in the limit of sharing information obtains the linear-optimal estimate.

Currently, work is being done towards the implementation of the consensus algorithm on the DFA with emphasis on robustness to communication link failure and asynchronous communication. An adequate filter will be integrated such that the proposed estimation method may be fused with other forms of attitude sensing for feedback control of the DFA.

Future work will focus on an investigation of the IR distance sensors' hardware limitations and on an automated calibration routine to improve estimation.



(a) Experiment set for an altitude of 10cm.



(b) Experiment set for an altitude of 20cm.

Fig. 8. The array is rotated by roll  $\gamma$  and pitch  $\beta$  separately at two different altitudes  $z$ . The reference angles are  $-10^\circ, -5^\circ, 5^\circ, 10^\circ$  and the reference altitude is 10cm, 20cm respectively. The states are estimated by combining all distance measurements of the array on one of the modules which is fully connected to the other modules. The mean of the estimated states is displayed in the respective figure together with the respective reference for each experiment.

## VII. ACKNOWLEDGEMENTS

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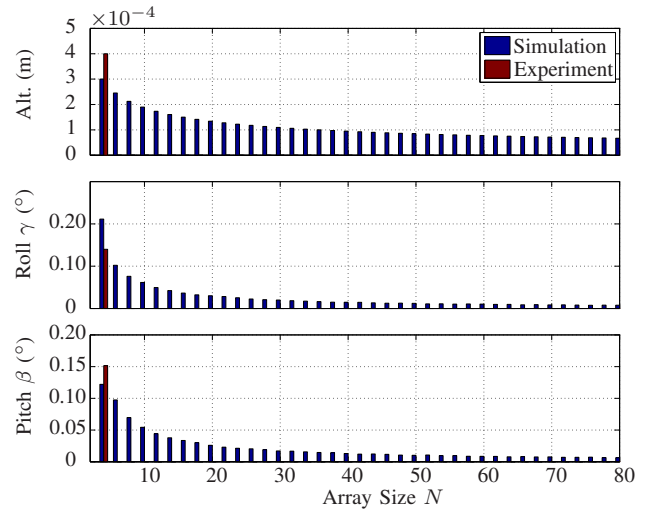


Fig. 9. The standard deviation of the state estimates from the experiment is of the same magnitude as the standard standard deviation of the simulated state estimates for the 4-module array. The standard deviation of the state estimates  $\hat{z}, \hat{\gamma}, \hat{\beta}$  decreases with the size of the simulated arrays which was expected from the results of Sec. III.

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